Rewarding Learning

ADVANCED SUBSIDIARY (AS)
General Certificate of Education
2024

Further Mathematics

Assessment Unit AS 2
assessing
Applied Mathematics

[SFM21]

FRIDAY 17 MAY, AFTERNOON

MARK SCHEME

GCE ADVANCED/ADVANCED SUBSIDIARY (AS) FURTHER MATHEMATICS

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right-hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for working.

MW indicates marks for combined method and working.

The solution to a question gains marks for correct method and marks for an accurate working based on this method. Where the method is not correct no marks can be given.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be followed through from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

Positive marking:

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of **following through** their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

(a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;

(b) readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

Section A

Mechanics 1

AVAILABLE MARKS

1 Let the extension of the vine be x

Final gravitational PE = GPE = $-5 \times 9.8 \times (8 + x) = -49\,(8 + x)$ M1 W1

Final elastic PE = EPE $= \dfrac{\lambda x^2}{2l} = \dfrac{128x^2}{16} = 8x^2$ M1 W1

Using Conservation of Mechanical Energy:
Taking the GPE baseline as the branch, initial GPE = 0
Initial EPE = 0
Both initial and final KEs are zero MW1

$KE_i + GPE_i + EPE_i = KE_f + GPE_f + EPE_f$

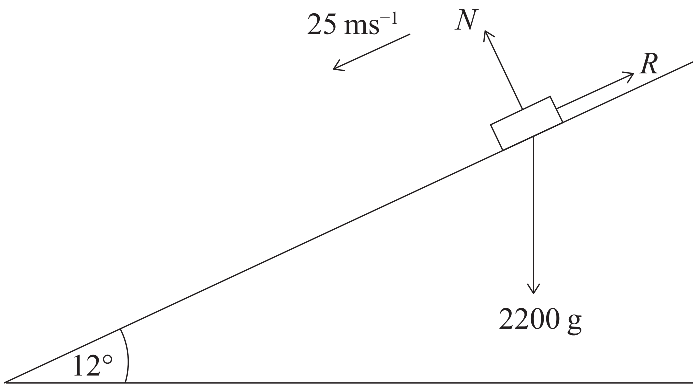
$0 = -49\,(8 + x) + 8x^2$ M1 W1

$8x^2 - 49x - 392 = 0$

$x = \dfrac{49 + \sqrt{49^2 + 4 \times 8 \times 392}}{16} = 10.7\text{m}$ W1 8

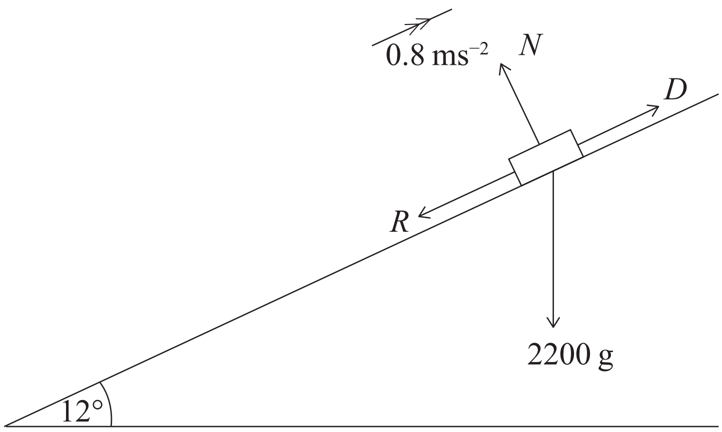
AVAILABLE MARKS

2 **(i)**



Resistance = Component of weight		
$R = 2200 \times 9.8 \times \sin 12°$	M1	
$= 4482.58$		
$= 4480$ N (3sf)	W1	
(ii) $R = 4482.58 = k \times 25^2$	M1	
$k = 7.172128$		
$= 7.17$ (3sf)	W1	

(iii)



$2200a = D - R - 2200 \times 9.8 \times \sin 12°$	M1 MW2	
When $v = 15\,\text{ms}^{-1}$,		
$R = 7.172128 \times 15^2 = 1613.73$	MW1	
$D = 2200 \times 0.8 + 1613.73 + 2200 \times 9.8 \times \sin 12° = 7856.3$	W1	
Power generated $= 7856.3 \times 15 = 117845$ W	M1	
$= 118$ kW (3sf)	W1	11

AVAILABLE MARKS

3 **(a)** Displacement is

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix} -1 \\ -2 \\ 4 \end{pmatrix} - \begin{pmatrix} -4 \\ 6 \\ \alpha \end{pmatrix} = \begin{pmatrix} 3 \\ -8 \\ 4-\alpha \end{pmatrix}$$ MW1

Work done = $\mathbf{F} \cdot (\overrightarrow{OB} - \overrightarrow{OA})$ M1

$$= \begin{pmatrix} 5 \\ 3 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -8 \\ 4-\alpha \end{pmatrix} = 15 - 24 - 2(4-\alpha)$$ MW1

$$= -17 + 2\alpha$$

No work is done, so $-17 + 2\alpha = 0$

$\alpha = 8.5$ MW1

(b) **(i)** Work done = $\int G \, dx$

$$= \int_1^4 \frac{20}{3}\left(3x - \frac{16}{x^3}\right) dx$$ M1 W1

$$= \left[10x^2 + \frac{160}{3x^2}\right]_1^4$$ MW2

$$= \left(160 + \frac{10}{3}\right) - \left(10 + \frac{160}{3}\right)$$

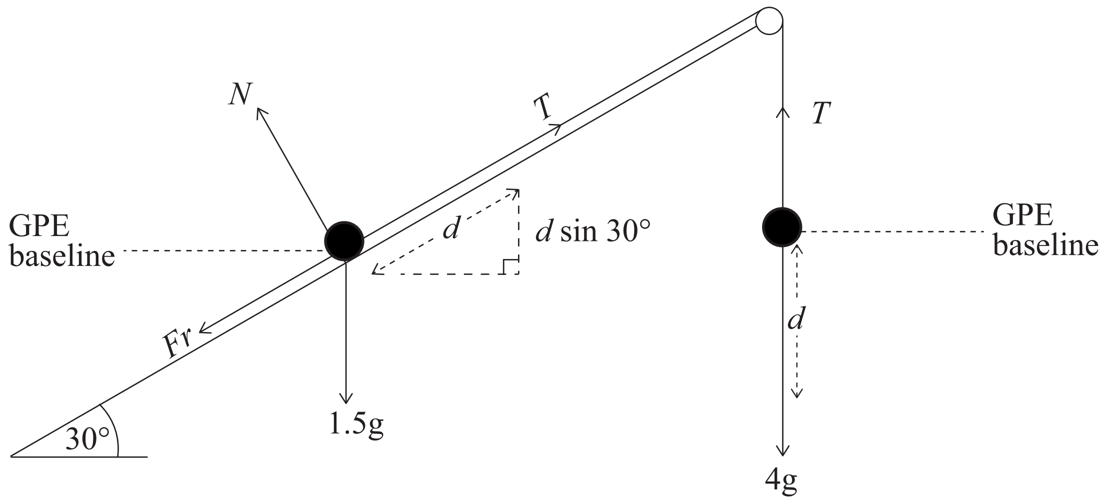
$$= 100 \text{ J}$$ W1

(ii) $\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \int G \, dx$ M1

$$\frac{1}{4}v^2 - \frac{441}{4} = 100$$

$v = 29 \text{ m s}^{-1}$ W1 11

4



(i) Gain in potential energy of bear $= m_{\text{bear}} \times g \times d \sin 30°$ M1

$= 1.5 \times 9.8 \times 0.4 \times 0.5$

$= 2.94\,\text{J}$ W1

Loss in potential energy of block $= m_{\text{block}}\, gd$

$= -4 \times 9.8 \times 0.4$

$= -15.68\,\text{J}$ MW1

Change in PE $= 2.94 - 15.68$

$= -12.74$

$= -12.7\,\text{J}$ (3sf) MW1

(ii) Gain in kinetic energy $= \frac{1}{2} m_{\text{bear}} \times v^2 + \frac{1}{2} m_{\text{block}} \times v^2$

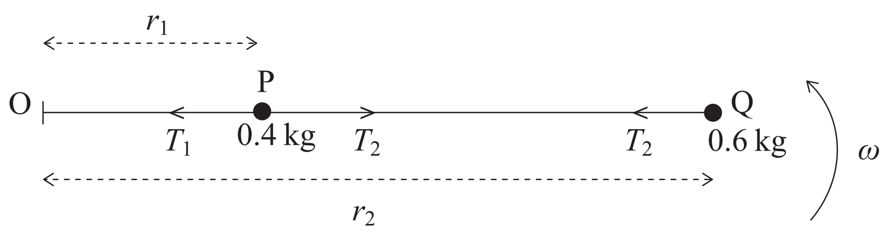
$= \frac{1}{2} \times 5.5 \times v^2 = 2.75v^2\,\text{J}$ M1 W1

(iii) By Work – Energy Principle,

$2.75v^2 - 12.74 = -1.7$ M1 W1

$v = 2.00\,\text{m s}^{-1}$ W1 9

5



		AVAILABLE MARKS
(i) Consider particle Q Newton's 2nd Law radially: $T_2 = m_2\,\omega^2\,r_2$		
$1.68 = 0.6\omega^2\,r_2$	M2	
$\omega^2\,r_2 = 2.8$	W1	
(ii) Consider particle P Newton's 2nd Law radially: $T_1 - T_2 = m_1\,\omega^2 r_1$		
$2.16 - 1.68 = 0.4\omega^2 r_1$	M2 W1	
$\omega^2 r_1 = 1.2$	W1	
(iii) $\dfrac{r_2}{r_1} = \dfrac{\omega^2 r_2}{\omega^2 r_1}$	M1	
$= \dfrac{2.8}{1.2} = \dfrac{7}{3}$	W1	
$v_2 = \omega r_2$	M1	
$= \dfrac{r_2}{r_1} \times \omega r_1$		
$= \dfrac{r_2}{r_1} \times v_1$		
$= \dfrac{7}{3} \times 1.5$		
$= 3.5\,\text{ms}^{-1}$	W1	11
	Section A	**50**

Section B

Mechanics

AVAILABLE MARKS

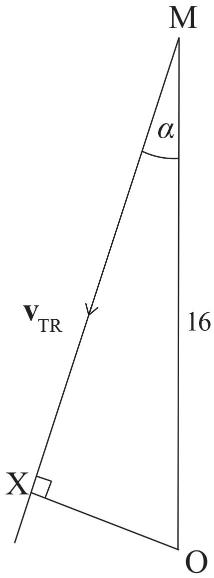
1 **(i)** $\mathbf{v}_R$ has direction $(24\mathbf{i} + 7\mathbf{j})$ and magnitude 5

Thus, $\mathbf{v}_R = \frac{5}{25} \times (24\mathbf{i} + 7\mathbf{j}) = (4.8\mathbf{i} + 1.4\mathbf{j})\ \text{m s}^{-1}$ M1 W1

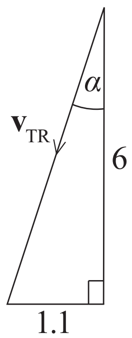
(ii) $\mathbf{v}_{TR} = \mathbf{v}_T - \mathbf{v}_R$

$= (3.7\mathbf{i} - 4.6\mathbf{j}) - (4.8\mathbf{i} + 1.4\mathbf{j}) = (-1.1\mathbf{i} - 6\mathbf{j})\ \text{m s}^{-1}$ M1 W1

(iii)



Displacement



Velocity

Consider motion relative to the Rabbit, ie Rabbit is stationary and Tractor moves with $\mathbf{v}_{TR}$ M1

Correctly plotted displacement diagram with the direction of $\mathbf{v}_{TR}$ plotted M1

OX = Perpendicular dropped from O to track of $\mathbf{v}_{TR}$ W1

$\tan \alpha = \frac{1.1}{6}$ MW1

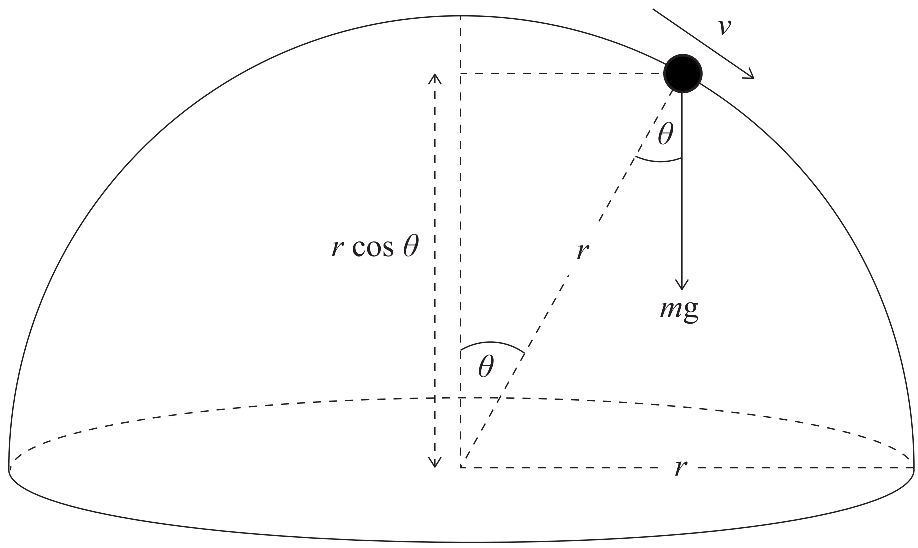
$\alpha = 10.388\ldots$

Shortest distance = OX = $16 \sin \alpha$ M1

$= 2.89\ \text{m}$ W1 10

		AVAILABLE MARKS
2 Let the mass of the satellite be m kg. Let the mass of the Earth be M kg and its radius be R metres. The Universal Gravitational Constant is G.		
On the Earth's surface, $mg = \dfrac{GMm}{R^2}$ so $GM = gR^2$	M1 W1	
At height h above the Earth's surface: $\dfrac{mv^2}{R+h} = \dfrac{GMm}{(R+h)^2}$	MW2 W1	
$v^2 = \dfrac{GM}{R+h}$		
$v = \sqrt{\dfrac{9.8 \times (6.37 \times 10^6)^2}{6.37 \times 10^6 + 6 \times 10^5}} = 7553.3$		
$= 7550 \text{ ms}^{-1}$ (3sf)	W1	6
3 **(i)** Taking dimensions $[x][k] = [\omega][t]$		
$L[k] = T^{-1}T$	MW1	
$[k] = L^{-1}$	W1	
(ii) Dimensions of LHS $= [\omega^2] = T^{-2}$		
Dimensions of RHS $= LT^{-2}L^{-1} = T^{-2}$	M1	
Both sides are equal therefore dimensionally equivalent	W1	
(iii) Taking dimensions $[\omega]^2 = [\rho]^a [g]^b [k]^c [\gamma]^d$	MW1	
Substituting dimensions $T^{-2} = (ML^{-3})^a(LT^{-2})^b(L^{-1})^c(MT^{-2})^d$	M1 W1	
Comparing indices M: $0 = a + d$	M1	
L: $0 = -3a + b - c$		
T: $-2 = -2b - 2d$	W1	
Solving in terms of d: $a = -d$ $b = 1-d$ $c = 1 + 2d$	M1 W1	
(iv) $\omega^2 \propto \rho^{-d} g^{1-d} k^{1+2d} \gamma^d$	M1	
$\omega^2 \propto gk\left(\dfrac{\gamma k^2}{\rho g}\right)^d$		
As the dimensions of ω^2 and gk are equivalent, this means $\dfrac{\gamma k^2}{\rho g}$ must be dimensionless.	W1	13

4



Let the mass of the particle be m kg.

Assume the particle leaves the hemisphere when the angle with the vertical is θ.

At that moment the normal reaction between the particle and the hemisphere will become zero.

Let the velocity of the particle be v at that moment.

Newton's 2nd law radially: $\dfrac{mv^2}{r} = mg\cos\theta$ M1 W2

So $v^2 = rg\cos\theta$ W1

Assuming GPE at 'centre' of hemisphere = 0

$$mgr + 0 = \frac{1}{2}mv^2 + mgr\cos\theta \qquad \text{M1 MW2}$$

$$mgr = \frac{1}{2}m(gr\cos\theta) + mgr\cos\theta$$

Alternative solution

assuming GPE at top of hemisphere = 0

$$0 = \frac{1}{2}mv^2 - mgr(1-\cos\theta) \qquad \text{M1 MW2}$$

$$0 = \frac{1}{2}m(gr\cos\theta) - mgr(1-\cos\theta)$$

Both of which lead to

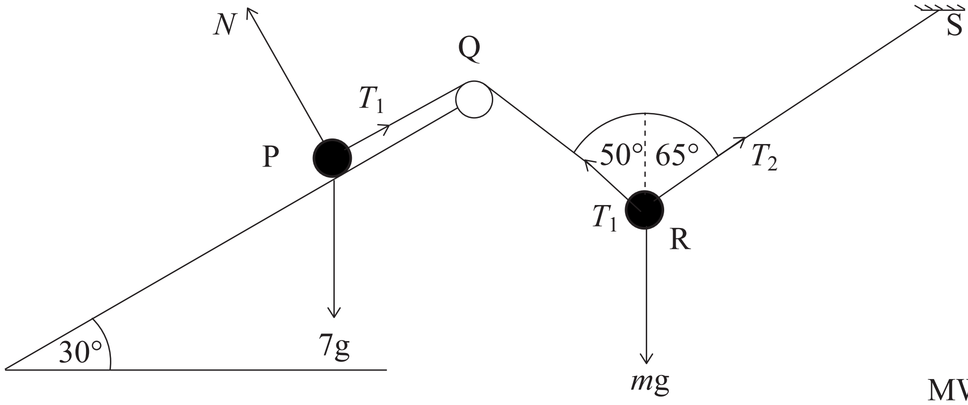
$$mgr = \frac{3}{2}mgr\cos\theta \qquad \text{W1}$$

$$\cos\theta = \frac{2}{3}$$

Distance travelled is $0.12 \times \cos^{-1}\left(\frac{2}{3}\right) = 0.101$ m MW1

AVAILABLE MARKS: 9

5 **(i)**



MW2

(ii) Resolving parallel to the plane at P: $T_1 = 7g \sin 30°$
$= 34.3\text{N}$ M1 W1

Let the extension in RS be x m.

Tension in RS: By Hooke's Law $T_2 = \dfrac{98x}{2.8} = 35x$ M1 W1

Resolving horizontally at R:
$T_1 \sin 50° = T_2 \sin 65°$ M1 W1

$34.3 \sin 50° = 35x \sin 65°$

$$x = \frac{34.3 \sin 50°}{35 \sin 65°}$$

$x = 0.828$ m W1

(iii) Resolving vertically at R:
$34.3 \times \cos 50° + 28.99 \times \cos 65° = mg$ M1 W1
$m = 3.5$ W1 12

Section B **50**

	AVAILABLE MARKS

Section C

Statistics

1 **(i)** Opportunity sampling MW1

(ii) Advantage: Cheap/straightforward/convenient MW1
Disadvantage: Introduces selection bias; therefore sample may not be representative of the population MW1

(iii) Members of the population are not equally likely of being selected for the sample. MW1 — 4

2 **(i)** $\sum p(x) = 1$

$a = 0.4$ MW1

(ii) $E(X) = 0$

$-1.4 + 0.2b + 0.1c + 0.7 + 0.7 = 0$

$0.2b + 0.1c = 0$ — ① M1W1

$E(X^2) = 12.85$

$0.4 \times (-3.5)^2 + 0.2b^2 + 0.1c^2 + 0.2 \times (3.5)^2 + 0.1 \times (7)^2 = 12.85$

$0.2b^2 + 0.1c^2 = 0.6$ — ② M1W1

Sub $c = -2b$ into ② gives $b^2 = 1$ M1

b is a negative integer $\therefore$ $b = -1$ and $c = 2$ MW1

(iii) $\text{Var}(X) = E(X^2) - [E(X)]^2$
$= 12.85 - 0^2 = 12.85$ MW1

(iv) $E(Y) = E(0.2X - 2.1) = 0.2\,E(X) - 2.1$
$= 0.2 \times 0 - 2.1 = -2.1$ MW1

$\text{Var}(Y) = \text{Var}(0.2X - 2.1) = 0.2^2\,\text{Var}(X)$
$= 0.2^2 \times 12.85 = 0.514$ M1

$\sigma(Y) = 0.717$ W1 — 11

AVAILABLE MARKS

3 **(i)** $S_{xx} = \sum x^2 - \frac{\left(\sum x\right)^2}{n} = 55 - \frac{225}{5} = 10$ M1W1

$b = \frac{S_{xy}}{S_{xx}}$

$-10.3 = \frac{S_{xy}}{10}$ M1

$S_{xy} = -103$ W1

$S_{xy} = \sum xy - \frac{\left(\sum x\right)\left(\sum y\right)}{n}$

$-103 = (715 + 4q) - \frac{15\left(292 + q\right)}{5}$ M1W1

$290 = 5q$
$q = 58$ W1

(ii) If the height increases by 1 km, the air pressure decreases by 10.3 kPa. MW2

(iii) $a = \bar{y} - b\,\bar{x}$

$\bar{y} = \frac{350}{5} = 70$ and $\bar{x} = 3$ M1W1

$a = 70 - (-10.3 \times 3)$
$a = 100.9$

$y = 100.9 - 10.3x$ MW1

When $x = 2.4$

$y = 100.9 - 10.3 \times 2.4$

Air pressure is 76.2 kPa (3sf) W1

(iv) Since 2.4 lies within the range of recorded values of x then this estimate should be reliable. W1 14

4 **(i)** Accept any **two** from the following
Mistakes occur randomly
Mistakes occur independently
Mistakes cannot occur simultaneously in a given interval of space or time
The mean number of mistakes is known, finite and in proportion to the given interval of space — MW2

(ii) X is the random variable 'number of incorrectly identified words on a page of text'

$X \sim \text{Po}(4.5)$ i.e. $\lambda = 4.5$

$P(X \geqslant 8) = 1 - P(X \leqslant 7)$ — M1
$= 1 - 0.9134$ — M1
$= 0.0866$ — W1

(iii) $P(X > 3) < 0.2$ — M1
$1 - P(X \leqslant 3) < 0.2$
$P(X \leqslant 3) > 0.8$ — MW1

From tables $P(X \leqslant 3) = \begin{cases} 0.7576 \text{ when } \lambda = 2.5 \\ 0.8571 \text{ when } \lambda = 2.0 \end{cases}$ — M1

$\therefore \ \lambda = 2$ — W1

AVAILABLE MARKS: 9

5 **(a)** **(i)** ${}^{14}C_6 = 3003$ — M1W1

(ii) (4USA × 2 members from 8 non-USA)

$= {}^6C_4 \times {}^8C_2$ — MW2

$= 420$ — MW1

Alternative solution

(4USA × 1China × 1Euro) + (4USA × 2China) + (4USA × 2Euro)

$= ({}^6C_4 \times {}^5C_1 \times {}^3C_1) + ({}^6C_4 \times {}^5C_2) + ({}^6C_4 \times {}^3C_2)$ — MW2

$= 225 + 150 + 45$

$= 420$ — MW1

(b) **(i)** $\dfrac{9!}{3!\,2!\,2!} = 15120$ — MW2W1

(ii) $\dfrac{7!}{3!\,2!} \times \dfrac{3!}{2!} = 1260$ — M1 MW1W1

P(three vowels in a group) $= \dfrac{1260}{15120} = \dfrac{1}{12}$ — MW1

AVAILABLE MARKS: 12

Section C — **50**

Section D

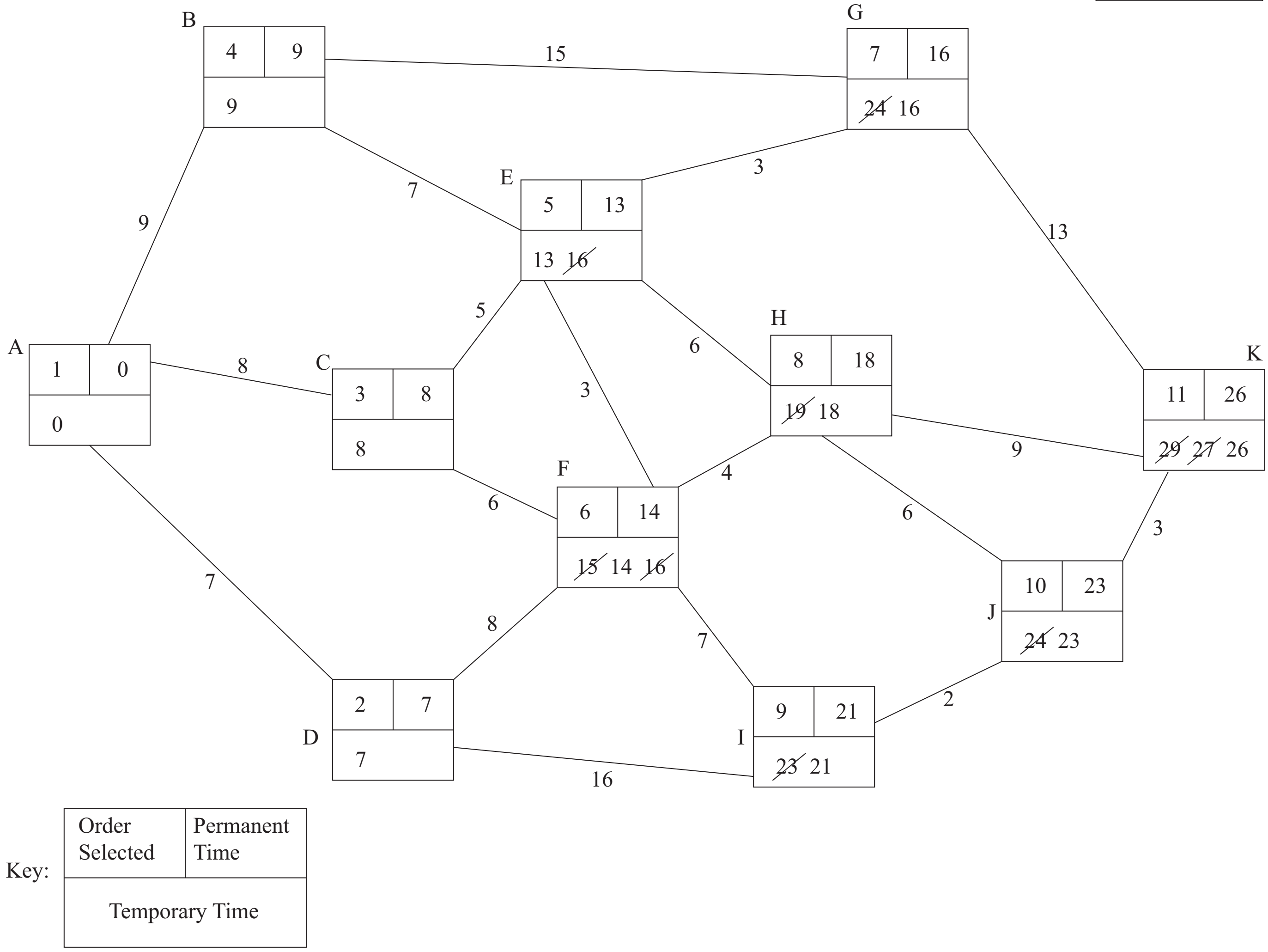
Discrete and Decision Mathematics

				AVAILABLE MARKS
1	**(i)**	Edges in list not repeated	M1	
		A full Eulerian circuit eg: e→a→f→d→e→b→f→c→d→a→b→c→e	W1	
	(ii)	K_3 subgraph of G	MW1	
		K_4, K_5 not subgraphs of G	MW2	
	(iii)	[ef], [ac] and [bd]	M1 W1	7
2	**(i)**	By inspection, $x^3 - 6x^2 + 11x - 6 = (x - 1)(x^2 - 5x + 6)$	M1 W1	
	(ii)	Substitute $u_n = kx^n$	M1	
		Obtaining $kx^{n+3} = 6kx^{n+2} - 11kx^{n+1} + 6kx^n$		
		$kx^n (x^3 - 6x^2 + 11x - 6) = 0$		
		$kx^n (x - 1) (x^2 - 5x + 6) = 0$	MW1	
		$x = 1$ or $(x - 2)(x - 3) = 0$		
		$x = 1$ or $x = 2$ or $x = 3$	MW1	
		$u_n = 2^n P + 3^n Q + R$	M1	
		$n = 0 \quad 6 = P + Q + R$	M1	
		$n = 1 \quad 7 = 2P + 3Q + R$		
		$n = 2 \quad 11 = 4P + 9Q + R$		
		Merging equations in pairs:		
		$1 = P + 2Q$		
		$4 = 2P + 6Q$		
		so $P = -1$, $Q = 1$ and $R = 6$	W1	
		$u_n = 3^n - 2^n + 6$	W1	
	(iii)	$n = 14$		
		$u_{14} = 3^{14} - 2^{14} + 6 = 4\ 766\ 591$	MW1	10

3 (a) (i) A B C D E H I F G — MW1

(ii) A B D E F G C H I — MW1

(b) (i)



Attempting to keep smallest temporary times	M1
Permanent times for A, B, C, and D correct	MW1
Permanent times for E and F correct	MW1
Permanent times for G, H, I and correct (allowing follow on)	MW1
Permanent times for J and K correct	MW1
(ii) Route A→C→F→I→J→K	M1 W1
(iii) 26 minutes	MW1

AVAILABLE MARKS: 10

4 **(i)**

p	p nand p
T	F
F	T

M1W1

AVAILABLE MARKS

(ii)

p	q	r	p nand q	p nand r	(p nand q) nand (p nand r)	q or r	p and (q or r)
T	T	T	F	F	T	T	T
T	T	F	F	T	T	T	T
T	F	T	T	F	T	T	T
T	F	F	T	T	F	F	F
F	T	T	T	T	F	T	F
F	T	F	T	T	F	T	F
F	F	T	T	T	F	T	F
F	F	F	T	T	F	F	F
a	a	a	b	c	d	e	g

a — M1 W1

b, c, d, e, g — MW5

conclusion d = g — M1 — 10

5 **(i)**

i	r	r^2	r^3	r^4	r^5	r^6	r^7
1	8	4	8	2	8	4	8

MW2

(ii) $\{i\}$, $\{i, r^4\}$, $\{i, r^2, r^4, r^6\}$, $\{i, r, r^2, r^3, r^4, r^5, r^6, r^7\}$ — M1 MW2

(iii) Subgroup of C_8 is $\{ i, r^2, r^4, r^6\}$

Subgroup of C_{12} is $\{i,\omega^3,\omega^6,\omega^9\}$ — MW1

Isomorphism $i\leftrightarrow i$ $\quad r^2 \leftrightarrow \omega^3$ $\quad r^4\leftrightarrow\omega^6$ $\quad r^6\leftrightarrow\omega^9$ or — M1 MW1

$i\leftrightarrow i$ $\quad r^2 \leftrightarrow \omega^9$ $\quad r^4\leftrightarrow\omega^6$ $\quad r^6\leftrightarrow\omega^3$

(iv)

$\times_{16}$	1	7	9	15
1	1	7	9	15
7	7	1	15	9
9	9	15	1	7
15	15	9	7	1

MW2

(v) All 3 non-identity elements in $(G, \times_{16})$ have period 2 — MW1

Only one element in C_8 (or C_{12}) has period 2, therefore — MW1

$(G, \times_{16})$ is not isomorphic to the four-element subgroups of C_8 and C_{12} — MW1 — 13

Section D — **50**